

Image of another universe being observed through a wormhole throat

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Abstract

We consider a technique of calculating deflection of the light passing through wormholes (from one universe to another). We find fundamental and characteristic features of electromagnetic radiation passing through the wormholes. Making use of this, we propose new methods of observing distinctive differences between wormholes and other objects as well as methods of determining characteristic parameters for different wormhole models.

1 Introduction

Lately, relativistic astrophysics has seen an increasing interest in papers where solutions with traversable wormholes (WHs) are discussed. This interest is caused, among other things, by projecting and constructing high-precision radiointerferometers which will make it possible to discriminate WHs from black holes.

In this paper we consider a static and spherically symmetric WH solution which only slightly differs from that for the extremal charged Reissner-Nordström black hole (see [1] or [2]). As is shown in [3], if the equation-of-state parameters of matter change their values the WH solution can smoothly turn into the extremal Reissner-Nordström solution with a horizon and the WH stops being traversable.

WH sustains its exotic properties thanks to phantom matter violating the null energy condition² and surrounding a WH's throat – see, for example, [4] or [3].

What can be practically interesting is the WH that only slightly differs from the extremally (whether electrically or magnetically) charged Reissner-Nordström black hole with the charge q . In this particular case the amount of the phantom matter can be taken arbitrarily small.

2 The Einstein equations

We take the metric tensor for a static and spherically symmetric WH in the following form:

$$ds^2 = e^{2\phi(r)} \cdot [dt^2] - e^{\lambda(r)} \cdot [dr^2] - r^2 \cdot ([d\theta^2] + \sin^2 \theta [d\varphi^2]) . \quad (1)$$

Let us consider the matter with a linear equation-of-state where the relation between the energy density ε and both the longitudinal (along radius) and transverse (perpendicular to radius) pressures (p_{\parallel} and p_{\perp} , respectively) is determined by a constant factor $1 + \delta$:

$$1 + \delta = -p_{\parallel}/\varepsilon = p_{\perp}/\varepsilon . \quad (2)$$

Using the notations

$$x = r/q, \quad e^{-\lambda} \equiv 1 - \frac{y(x)}{x}, \quad \xi(x) \equiv 8\pi\varepsilon q^2, \quad z(x) \equiv (1 - 1/x) . \quad (3)$$

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² This condition means satisfying the following inequality (NEC): $T_{ik}\zeta^i\zeta^k > 0$, T_{ik} being the energy-momentum tensor of the matter and ζ^i – the null 4-vector of photon. Physically, the exoticness of the phantom matter violating the NEC leads to the possibility of choosing a reference frame where observed energy density is negative.

and introducing the prime for derivatives with respect to x allows one to write down the Einstein equations³:

$$8\pi\epsilon r^2 = \xi x^2 = -e^{-\lambda} (1 - x\lambda') + 1, \quad (4)$$

$$8\pi p_{\parallel} r^2 = -(1 + \delta)\xi x^2 = e^{-\lambda} (1 + 2x\phi') - 1, \quad (5)$$

$$8\pi p_{\perp} r^2 = (1 + \delta)\xi x^2 = e^{-\lambda} \left(x^2 \phi'' + x^2 \phi'^2 - x\lambda'/2 - x^2 \phi' \lambda'/2 + x\phi' \right). \quad (6)$$

The Einstein equations comprise the Bianchi identities which in this case have the form:

$$(1 + \delta)(\ln \xi)' + 4(1 + \delta)(\ln x)' + \delta\phi' = 0 \quad (7)$$

Denoting $x_0 \equiv y(x_0)$ the coordinate of the WH throat, we represent eqs. (4), (5), and (7), in the following convenient form:

$$y(x) = x_0 + \int_{x_0}^x \xi x^2 dx \quad (8)$$

$$\left(\ln [\xi x^4] \right)' = \frac{\delta}{2(1 + \delta)} \cdot \frac{\xi x(1 + \delta) - y/x^2}{1 - y/x} \quad (9)$$

$$\exp[\phi(x)] = [\xi x^4]^{-(1+\delta)/\delta} \quad (10)$$

If $\delta = 0$, this solution turns into the extremal Reissner-Nordström solution which has a horizon:

$$y_{[\delta=0]}(x) = 2 - 1/x, \quad \xi_{[\delta=0]}(x) = 1/x^4, \quad \exp[\phi_{[\delta=0]}(x)] = 1 - 1/x. \quad (11)$$

Let us obtain the explicit analytical form of the solution to the first order in the small correction δ . In the linear approximation with respect to δ eq. (9) takes the form:

$$\frac{\partial \ln(\xi x^4)}{\partial x} = -\delta \cdot \frac{\partial \ln(z)}{\partial x} \quad (12)$$

Taking into account that, at infinity, the value of ξ must be equal to that of $\xi_{[\delta=0]}$, we obtain the approximate solution:

$$\xi x^4 = z^{-\delta}, \quad y(x) = x_0 + \frac{z^{1-\delta} - z_0^{1-\delta}}{1 - \delta} \quad \left(z_0 \equiv 1 - \frac{1}{x_0} \right), \quad \exp(\phi) = z^{1+\delta}. \quad (13)$$

The relation between the throat radius x_0 and δ follows from the fact that the asymptotics of the functions $e^{-\lambda}$ and $e^{2\phi}$ must become equal as $x \rightarrow \infty$:

$$x_0 + \frac{1 - z_0^{1-\delta}}{1 - \delta} \rightarrow 2(1 + \delta) \quad (14)$$

This is a transcendental equation yielding the throat radius. The detailed analysis of the equation gives the asymptotics:

$$\lim_{\delta \rightarrow 0} \delta = (x_0 - 1)^2, \quad (15)$$

with the parameter δ being defined by the equation-of-state of the matter in the WH.

Strictly speaking, the expansion into a series with respect to δ is not quite well-posed. A straightforward substitution of relations (13) in eqs. (5) and (6) clearly demonstrates this. But the WH solution can be looked for in an inverse manner, viz. first, one can use expressions (13) as the WH metrics components $e^{2\phi}$ and e^{λ} and then find expression for p_{\parallel} and p_{\perp} from eqs. (5) and (6). This last approach is no worse defined than just solving the Einstein equations.

³Their derivation is available, for example, in [5] (task 5 to §100).

3 Light passing through the throat

Let the other universe contain \mathbf{N} stars with equal luminosities and suppose $\mathbf{N} \gg \mathbf{1}$. Let all the stars be homogeneously distributed over the celestial sphere in the other universe.

An observer in our Universe who is looking at the stars in the other universe through the WH throat sees them inhomogeneously distributed over the throat. This is because of the fact that the WH throat refracts and distorts the light of these stars. The distortion will obviously be spherically symmetric with the throat center being the symmetry center.

Now let the observer look only at the fraction of the stars seen in the thin ring with the center coinciding with the throat center, the ring radius being \mathbf{h} and its width $-\mathbf{d}\mathbf{h}$. Hence, the observer surveys the solid angle $\mathbf{d}\Omega$ of the other universe and, moreover, $\mathbf{d}\Omega = 2\pi |\sin \theta| d\theta$. Here, $\theta(\mathbf{h})$ is the deflection angle of light rays passing through the WH throat measured relative to the rectilinear propagation⁴. Since the total solid angle equals 4π , the observer can see $\mathbf{dN} = \mathbf{N} \mathbf{d}\Omega / (4\pi)$ stars in the ring⁵. Furthermore, the apparent density of the stars (per unit area of the ring $\mathbf{dS} = 2\pi \mathbf{h} \mathbf{d}\mathbf{h}$) is $\mathbf{J} = \mathbf{dN} / \mathbf{dS}$. We, therefore, obtain:

$$J(h) = \frac{N |\sin \theta|}{4\pi h} \cdot \frac{d\theta}{dh} \quad (16)$$

In the paper [3] the dependance $\theta(h)$ was obtained:

$$\theta(h) = 2 \int_0^{1/x_0} \frac{\tilde{h}}{\sqrt{(1-\eta y)(e^{-2\phi} - \eta^2 \tilde{h}^2)}} d\eta, \quad (17)$$

where the notations $\eta \equiv 1/x$ and $\tilde{h} \equiv h/q$ were used. This yields

$$\frac{d\theta}{dh} = \frac{2}{q} \int_0^{1/x_0} \frac{e^{-2\phi}}{\sqrt{1-\eta y} \cdot [e^{-2\phi} - \eta^2 \tilde{h}^2]^{3/2}} d\eta. \quad (18)$$

Taking advantage of formulae (13), (16), (17), and (19), one can find the expression for the apparent density of the stars $\mathbf{J}(\mathbf{h})$ in the wormhole.

Note that formula (17) also gives the maximum possible impact parameter $\mathbf{h} = \mathbf{h}_{\max}$ which still allows the observer to see the stars of the other universe. This parameter corresponds to a zero of the second factor in the radicand in (17). Namely, \mathbf{h}_{\max} is equal to the least possible value of the function $e^{-\phi}/\eta$. Having conducted trivial inquiry, we obtain

$$\tilde{h}_{\max} = 4 \cdot 2^\delta \approx 4. \quad (19)$$

as $\delta \rightarrow 0$.

The distortion of the light rays that had passed through the WH throat is caused not only by re-distributing of the star density, but also by changes in their apparent brightness. Namely, as the impact parameter \mathbf{h} increases the stellar brightness changes. This is because of the fact that as the radius \mathbf{h} of the ring, through which the star light passes, increases, an element of the solid angle where this light scatters changes as well. The respective change in the stellar brightness is proportional to the quantity $\kappa = \mathbf{dS} / \mathbf{d}\Omega$. Therefore, the total brightness of all the stars seen on unit area of the above-mentioned ring is $\mathbf{dN} \cdot \kappa / \mathbf{dS}$.

⁴By convention, the rectilinear propagation means the trajectory passing through the center of the WH throat.

⁵Since the light deflection angle θ can exceed π , the total solid angle turns out to be more than 4π . This change, however, reduces to another constant (instead of 4π) and does not affect the final result.

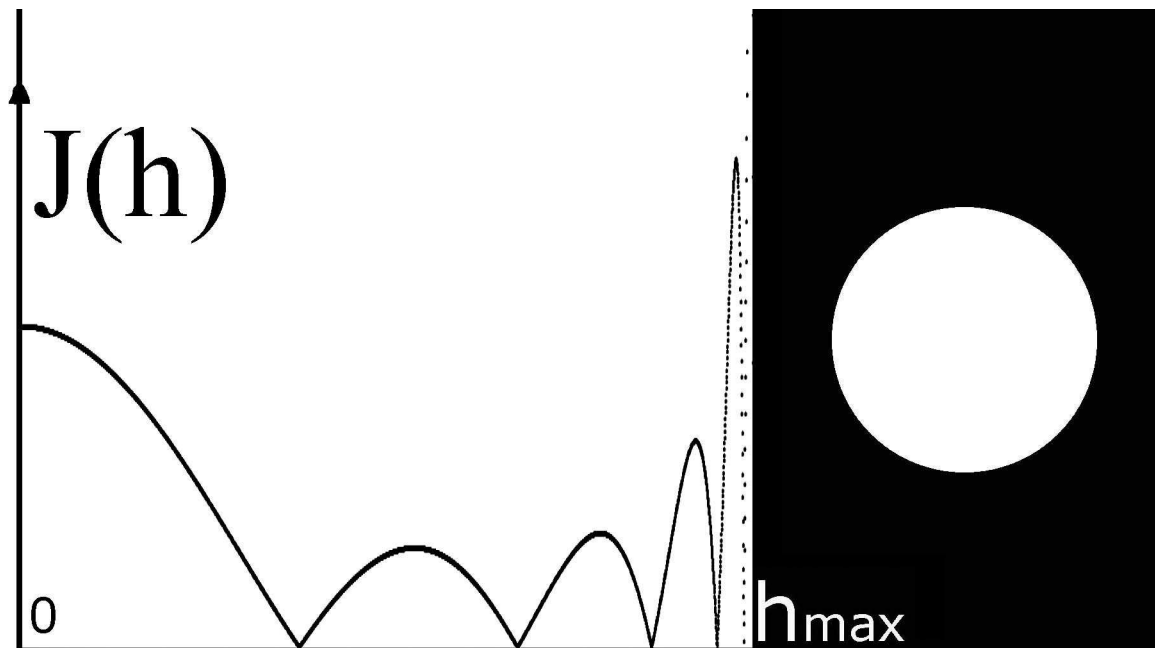


Figure 1: The left panel shows the dependance $J(h)$ when $h \in (0, h_{max})$ and $\delta = 0.001$ while the right one is the apparent image of the sky of the other universe as being seen through the WH throat as $N \rightarrow \infty$.

Thus, we obtain that as $N \rightarrow \infty$ the apparent brightness of the WH's part inside its throat does not depend on impact parameter and, regardless of which WH model we use, the WH looks like a homogeneous spot in every wavelength range.

We point out in this regard that the conclusion in the paper [6] about inhomogeneous observation of the light from the other universe is false. In that article correct mathematical expressions were misinterpreted.

4 Conclusion

In spite the result obtained stating that the light distribution in the WH throat is homogeneous for each WH model, it is worth noting that in the real universe the number of visible stars is finite, though big. This implies that if angular resolution of the observer's instrument in our Universe is high enough they will be able to discover the changing star density in the throat $\mathbf{J}(\mathbf{h})$. The left panel of Fig. 1 shows this plot for $\delta = 0.001$. Sharp minima on the plot correspond to zeros of the sine in expression (16). This is because at sufficiently large impact parameters the light rays are deflected by large angles ($\theta > \pi$) so that in the vicinities of the points $\theta = \pi n$ abrupt declines in distribution arise. But near these declines the observed stellar brightness tends to infinity (lensing), which ultimately provides the (on average) uniform light flow over the WH throat (see the right panel of Fig. 1).

Positions of the declines depend on the value of δ . Hence, registering them makes it possible to determine the equation-of-state parameters of the WH matter and features of the WH model (which is highly analogous to processing the light spectra).

Thus, in this paper we have proposed a technique of calculating the deflection of light passing through wormholes as well as methods of observing distinctive features of specific WH models.

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